Fast and accurate floating-point division on ST231
Algorithm, implementation, and automatic generation and validation

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Context and objectives

Context

► FLIP software library
  → http://flip.gforge.inria.fr/
  → support for floating-point arithmetic on integer processors

► low latency implementation of binary floating-point division
  → targets a VLIW integer processor of the ST200 family

► no support of subnormal numbers
  → input/output: ±0, ±∞, NaN or normal number

Objectives

► faster software implementation (compared to FLIP 0.3)
  → expose instruction-level parallelism via bivariate polynomial evaluation

► correctly rounded
  → rounding-to-nearest even
IEEE 754 specification

Let \((x, y)\) be two binary floating-point data:

\[
x/y = (-1)^{s_r} \cdot |x|/|y|,
\]

with \(s_r = s_x \text{ XOR } s_y\).

| \(|x|/|y|\) | \(|y|\) |
|---|---|---|---|
| +0 normal | +0 | +0 qNaN |
| +∞ RN\(_p\)(\(|x|/|y|\)) | +0 qNaN |
| +∞ +∞ | qNaN qNaN |
| NaN qNaN qNaN qNaN |

Special values for RN\(_p\)(\(|x|/|y|\)).

⇒ since RN\(_p\)(\(-r\)) = -RN\(_p\)(\(r\)), for non special inputs:

\[
\text{RN}_p(x/y) = (-1)^{s_r} \cdot \text{RN}_p(|x|/|y|).
\]
Notation and assumptions

- **Input** \((x, y)\): two positive normal numbers
  - \(\rightarrow\) precision \(p\), extremal exponents \((e_{\text{min}}, e_{\text{max}})\)

\[
x = (-1)^{s_x} \cdot m_x \cdot 2^{e_x}
\]

\[
\begin{align*}
  s_x & \in \{0, 1\} \\
  m_x & = 1.m_{x,1} \ldots m_{x,p-1} \in [1, 2) \\
  e_x & \in \{e_{\text{min}}, \ldots, e_{\text{max}}\}
\end{align*}
\]

- **Computation**: \(k\)-bit unsigned integers
  - \(\rightarrow\) register size \(k\)

- **Example for binary32 format**: \((k, p, e_{\text{max}}) = (32, 24, 127)\)
Outline of the talk

Division via polynomial evaluation

Generation of an efficient evaluation program

Validation of the generated evaluation program

Experimental results

Concluding remarks
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Division algorithm flowchart

- Definition

\[ c = \begin{cases} 
1 & \text{if } m_x \geq m_y, \\
0 & \text{if } m_x < m_y. 
\end{cases} \]
Division algorithm flowchart

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\[ c = \begin{cases} 
  1 & \text{if } m_x \geq m_y, \\
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\end{cases} \]

- **Range reduction**

\[
x/y = (2^{m_x/m_y} \cdot 2^{-c}) \times 2^{e_x - e_y - 1 + c}
\]

\[
\ell = 2^{m_x/m_y} \cdot 2^{-c}
\]

\(\ell \in [1, 2)\)

\[
\text{RN}_p(\ell) \in [1, 2)
\]

\[
\text{RN}_p(x/y) = \text{RN}_p(\ell) \times 2^d
\]
Division algorithm flowchart

- **Definition**

  \[
  c = \begin{cases} 
  1 & \text{if } m_x \geq m_y, \\
  0 & \text{if } m_x < m_y.
  \end{cases}
  \]

- **Range reduction**

  \[
  x/y = \left(\frac{2m_x}{m_y} \cdot 2^{-c}\right) \times 2^{e_x-e_y-1+c}
  \]

How to compute the correctly rounded significand \( \text{RN}_p(\ell) \)?
How to compute a correctly rounded significand?

- **Iterative methods** (restoring, non-restoring, ...)
  - Oberman and Flynn (1997)
  - minimal instruction-level parallelism exposure, sequential algorithm
How to compute a correctly rounded significand?

- **Iterative methods** *(restoring, non-restoring, ...)*
  - Oberman and Flynn (1997)
  - minimal instruction-level parallelism exposure, sequential algorithm

- **Multiplicative methods** *(Newton-Raphson, Goldschmidt)*
  - more instruction-level parallelism exposure
  - previous implementation of division (FLIP 0.3)
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  - more instruction-level parallelism exposure
  - previous implementation of division (FLIP 0.3)

- **Polynomial-based methods**
  - Agarwal, Gustavson and Schmookler (1999)
    → univariate polynomial evaluation
  - Our approach
    → single bivariate polynomial evaluation
Truncated one-sided approximation

- See for example, Ercegovac and Lang (2004)
- 3 steps
  1. compute \( v = (01.v_1 \ldots v_{k-2}) \) such that 
     \[ -2^{-p} \leq \ell - v < 0 \] 
     that is implied by 
     \[ |(\ell + 2^{-p-1}) - v| < 2^{-p-1} \]
  2. truncate \( v \) after \( p \) fraction bits: \( w = (01.v_1 \ldots v_p0\ldots0) \)
  3. obtain \( \text{RN}_p(\ell) \) after possibly adding \( 2^{-p} \)
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\[ w \]
\[ \ell \]

\[ w \]
\[ \ell \]

3. obtain \( \text{RN}_p(\ell) \) after possibly adding \( 2^{-p} \)

How to compute the one-sided approximation \( v \)?
Computation of the one-sided approximation

1. Consider $\ell + 2^{-p-1}$ as the exact result of the function

$$F(s, t) = s/(1 + t) + 2^{-p-1},$$

at the points $s^* = 2^{1-c}m_x$ and $t^* = m_y - 1$:

$$\ell + 2^{-p-1} = F(s^*, t^*).$$
Computation of the one-sided approximation

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   at the points \( s^* = 2^{1-c}m_x \) and \( t^* = m_y - 1 \):
   \[
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   \]

2. Approximate \( F(s, t) \) by a bivariate polynomial \( P(s, t) \)
   \[
   P(s, t) = s \cdot a(t) + 2^{-p-1}.
   \]

\( \rightarrow a(t) \): univariate polynomial approximant of \( 1/(1 + t) \)
\( \rightarrow \) approximation entails an error \( \epsilon_{\text{approx}} \)
Computation of the one-sided approximation

1. Consider $\ell + 2^{-p-1}$ as the exact result of the function
   
   $$F(s, t) = s/(1 + t) + 2^{-p-1},$$

   at the points $s^* = 2^{1-c}m_x$ and $t^* = m_y - 1$:
   
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2. Approximate $F(s, t)$ by a bivariate polynomial $P(s, t)$
   
   $$P(s, t) = s \cdot a(t) + 2^{-p-1}.$$

   → $a(t)$: univariate polynomial approximant of $1/(1 + t)$
   
   → approximation entails an error $\epsilon_{\text{approx}}$

3. Evaluate $P(s, t)$ by a well-chosen efficient evaluation program $\mathcal{P}$
   
   $$v = \mathcal{P}(s^*, t^*).$$

   → evaluation entails an error $\epsilon_{\text{eval}}$
Computation of the one-sided approximation

1. Consider $\ell + 2^{-p-1}$ as the exact result of the function

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F(s, t) = s/(1 + t) + 2^{-p-1},
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\]

$\rightarrow$ evaluation entails an error $\epsilon_{\text{eval}}$

How to ensure that $|((\ell + 2^{-p-1}) - v| < 2^{-p-1}$?
Sufficient error bounds

- Since by triangular inequality
  \[ |(\ell + 2^{-p-1}) - v| \leq \mu \cdot \epsilon_{\text{approx}} + \epsilon_{\text{eval}} \]
  with
  \[ \mu = \max\{s^*\} = \max\{2^{1-c}m_x\} = (4 - 2^{3-p}) \]
Sufficient error bounds

Since by triangular inequality

$$| (\ell + 2^{-p-1}) - v | \leq \mu \cdot \epsilon_{\text{approx}} + \epsilon_{\text{eval}}$$

with

$$\mu = \max \{ s^* \} = \max \{ 2^{1-c} m_x \} = (4 - 2^{3-p})$$

One has to ensure

$$\mu \cdot \epsilon_{\text{approx}} + \epsilon_{\text{eval}} < 2^{-p-1}$$

Sufficient conditions can be obtained

$$\epsilon_{\text{approx}} < 2^{-p-1}/\mu \quad \text{and} \quad \epsilon_{\text{eval}} < 2^{-p-1} - \mu \cdot \epsilon_{\text{approx}}$$
Implementation steps

1. determine the minimal degree $\delta$ for the polynomial approximant $a$

2. compute the polynomial approximant $a$ such that

$$\epsilon_{\text{approx}} < 2^{-p-1}/\mu$$

3. find an efficient evaluation program $P$ such that

$$\epsilon_{\text{eval}} < 2^{-p-1} - \mu \cdot \epsilon_{\text{approx}}$$

4. validate the evaluation program

$\Rightarrow$ implemented using Sollya (steps 1 and 2) and Gappa (step 4)
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Automatic generation of an efficient evaluation program

- Evaluation program \( P = \) main part of the full software implementation
  \( \rightarrow \) dominates the cost

- By \textit{efficient}, one means an evaluation program that
  \( \rightarrow \) reduces the evaluation latency
  \( \rightarrow \) reduces the number of multiplications
  \( \rightarrow \) is accurate enough
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- Target architecture: ST231
  - 4-issue VLIW integer processor with at most 2 mul. per cycle
  - latencies: addition = 1 cycle, multiplication = 3 cycles
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Which evaluation program to evaluate the polynomial $P(s, t)$?
Example for the binary32 implementation: \((k, p) = (32, 24)\)

\[ P(s, t) = 2^{-p-1} + s \cdot \sum_{i=0}^{10} a_i t^i \]

- Horner’s scheme: 
  \((3 + 1) \times 11 = 44\) cycles

  \(\rightarrow\) sequential scheme, no instruction-level parallelism exposure
Example for the binary32 implementation: \((k, p) = (32, 24)\)

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- **Horner’s scheme**: \((3 + 1) \times 11 = 44\) cycles
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- **Estrin’s scheme**: 20 cycles
  - more instruction-level parallelism
  - a last multiplication by \(s\)
  - 2 cycles save by distributing the multiplication by \(s\) in the evaluation of the univariate polynomial \(a(t)\)
Example for the binary32 implementation: \((k, p) = (32, 24)\)

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- ...  

We can do much better.

- But how to explore the solution space and choose an efficient evaluation program?
  - interest of automatic generation
Efficient evaluation tree generation

- Similar to Harrison, Kubaska, Story and Tang (1999)

- Assumption
  - unbounded parallelism
  - latencies of arithmetic operators: \(+\) and \(\times\)
Efficient evaluation tree generation

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  - latencies of arithmetic operators: $+$ and $\times$

- Two sub-steps
  1. determine a target latency $\tau$
     
     ie. $\tau = 3 \times \lceil \log_2(\deg(P)) \rceil + 1$
  2. generate automatically a set of evaluation trees, with height $\leq \tau$
Efficient evaluation tree generation

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\[ \Rightarrow \] if no tree satisfies \( \tau \) then increase \( \tau \) and restart
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  2. generate automatically a set of evaluation trees, with height $\leq \tau$

      $\Rightarrow$ if no tree satisfies $\tau$ then increase $\tau$ and restart

- Number of evaluation trees = extremely large $\rightarrow$ several filters
Efficient evaluation tree generation

\[ P(s, t) = 2^{-p-1} + s \sum_{i=0}^{10} a_i t^i \]

- first target latency \( \tau = 13 \)
  \[ \rightarrow \text{no tree found} \]
Efficient evaluation tree generation

\[ P(s, t) = 2^{-p-1} + s \cdot \sum_{i=0}^{10} a_i t^i \]

- first target latency \( \tau = 13 \)
  \( \rightarrow \) no tree found

- second target latency \( \tau = 14 \)
  \( \rightarrow \) obtained in about 10 sec.
Efficient evaluation tree generation

\[ P(s, t) = 2^{-p-1} + s \cdot \sum_{i=0}^{10} a_i t^i \]

- first target latency \( \tau = 13 \)
  \( \rightarrow \) no tree found

- second target latency \( \tau = 14 \)
  \( \rightarrow \) obtained in about 10 sec.

- distribute the multiplication by \( s \)
  \( \rightarrow \) otherwise: 18 cycles

- too difficult to find such tree by hand
Arithmetic operator choice

- Polynomial coefficients implemented in absolute value
- All intermediate values have constant sign
  ⇒ not store the sign: more accuracy
Arithmetic operator choice

- Polynomial coefficients implemented in absolute value
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- Label evaluation trees by appropriate arithmetic operator: + or −
Arithmetic operator choice

- Polynomial coefficients implemented in absolute value
- All intermediate values have constant sign
  - not store the sign: more accuracy

- Label evaluation trees by appropriate arithmetic operator: $+$ or $-$

- If the sign of an intermediate value changes when the input varies then the evaluation tree is rejected
  - implementation with certified interval arithmetic (MPFI)
Practical scheduling checking

- Schedule the evaluation trees on a simplified model of a real target architecture
  - operator costs, nb. issues, constraints on operators
  - no syllables constraint
Practical scheduling checking

- Schedule the evaluation trees on a simplified model of a real target architecture
  - operator costs, nb. issues, constraints on operators
  - no syllables constraint

- Check if no increase of latency in practice compared to the latency on unbounded parallelism
  \rightarrow if practical latency > theoretical latency then the evaluation tree is rejected
  \rightarrow implementation using naive list scheduling algorithm is enough
Outline of the talk

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Example for the binary32 implementation: \((k, p) = (32, 24)\)

- Approximation of \(1/(1 + t)\) by truncated Remez’ polynomial of degree 10

\[
\epsilon_{\text{approx}} \leq 2^{-27.41} \approx 6.0 \times 10^{-9} < 2^{-25}/(4 - 2^{-21}) \approx 7.4 \times 10^{-9}
\]
Example for the binary32 implementation: \((k, p) = (32, 24)\)

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- Deduction of the evaluation error bound from \(\epsilon_{\text{approx}}\)

\[
\epsilon_{\text{eval}} < 2^{-25} - (4 - 2^{-21}) \cdot 2^{-27.41\ldots} \approx 2^{-26.9999\ldots} \approx 7.4 \times 10^{-9}
\]
Example for the binary32 implementation: \((k, p) = (32, 24)\)

- Case 1: \(m_x \geq m_y \rightarrow \) condition satisfied
- Case 2: \(m_x < m_y \rightarrow \) condition not satisfied

ie. \(s^* = 3.935581684112548828125\) and \(t^* = 0.97490441799163818359375\)
Example for the binary32 implementation: \((k, p) = (32, 24)\)

- **Case 1:** \(m_x \geq m_y \rightarrow \) condition satisfied
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1. determine an interval \(I\) around this point
Example for the binary32 implementation: \((k, p) = (32, 24)\)

- **Case 1:** \(m_x \geq m_y \rightarrow \text{condition satisfied}
- **Case 2:** \(m_x < m_y \rightarrow \text{condition not satisfied}

ie. \(s^* = 3.935581684112548828125\) and \(t^* = 0.97490441799163818359375\)

![Graph showing approximation error and interval I around point t = my - 1]

1. determine an interval \(I\) around this point
2. compute \(\epsilon_{\text{approx}}\) over \(I\)
3. determine an evaluation error bound \(\eta\)
4. check if \(\epsilon_{\text{eval}} < \eta\)?
Evaluation program validation strategy

- Find a splitting of the input interval into $n$ subinterval(s) $\mathcal{T}^{(i)}$, and check that

$$\mu \cdot \epsilon^{(i)}_{\text{approx}} + \epsilon^{(i)}_{\text{eval}} < 2^{-p-1}$$

on each subinterval.
**Evaluation program validation strategy**

- Find a splitting of the input interval into \( n \) subinterval(s) \( \mathcal{T}^{(i)} \), and check that

\[
\mu \cdot \varepsilon^{(i)}_{\text{approx}} + \varepsilon^{(i)}_{\text{eval}} < 2^{-p-1}
\]

on each subinterval.

- Implementation of the splitting by **dichotomy**
  - for each \( \mathcal{T}^{(i)} \)
    1. compute a certified approximation error bound \( \varepsilon^{(i)}_{\text{approx}} \)
    2. determine an evaluation error bound \( \varepsilon^{(i)}_{\text{eval}} \)
    3. check this bound

\( \implies \) if this bound is not satisfied, \( \mathcal{T}^{(i)} \) is split up into 2 subintervals

- implemented using **Sollya** (steps 1 and 2) and **Gappa** (step 3)
Evaluation program validation strategy

- Find a splitting of the input interval into \( n \) subinterval(s) \( T^{(i)} \), and check that

\[
\mu \cdot \epsilon_{\text{approx}}^{(i)} + \epsilon_{\text{eval}}^{(i)} < 2^{-p-1}
\]

on each subinterval.

- Implementation of the splitting by dichotomy

  - for each \( T^{(i)} \)
    1. compute a certified approximation error bound \( \epsilon_{\text{approx}}^{(i)} \)
    2. determine an evaluation error bound \( \epsilon_{\text{eval}}^{(i)} \)
    3. check this bound

⇒ if this bound is not satisfied, \( T^{(i)} \) is split up into 2 subintervals
  - implemented using \textit{Sollya} (steps 1 and 2) and \textit{Gappa} (step 3)

- Example of binary32 implementation
  → launched on a 64 processor grid
  → 36127 subintervals found in several hours (≈ 5h.)
Evaluation program validation strategy

* Does the condition
\[ \mu \cdot \epsilon_{\text{approx}}^{(i)} + \epsilon_{\text{eval}}^{(i)} < 2^{-p-1} \]
hold for \( i \in \{1, \ldots, n\} \)?

<table>
<thead>
<tr>
<th>Depth</th>
<th>Subintervals</th>
<th>( \epsilon_{\text{approx}}(a) \leq \epsilon_{\text{eval}}(P) &lt; )</th>
<th>*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( l_{1,1} = [2^{-23}, 1 - 2^{-23}] )</td>
<td>( \theta_1 \approx 2^{-27.41} ) ( \eta_1 \approx 2^{-26.99} ) no</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( l_{2,1} = [2^{-23}, 0.5 - 2^{-23}] )</td>
<td>( \theta_2 \approx 2^{-27.41} ) ( \eta_2 \approx 2^{-26.99} ) no yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( l_{2,2} = [0.5, 1 - 2^{-23}] )</td>
<td>( \theta_1 \approx 2^{-27.41} ) ( \eta_1 \approx 2^{-26.99} ) no yes</td>
<td></td>
</tr>
<tr>
<td>( \ldots )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( j )</td>
<td>( l_{j,1} = [2^{-23}, 0.5 - 2^{-23}] )</td>
<td>( \theta_2 \approx 2^{-27.41} ) ( \eta_2 \approx 2^{-26.99} ) no yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( l_{j,2} = [0.5, 0.75 - 2^{-23}] )</td>
<td>( \theta_1 \approx 2^{-27.41} ) ( \eta_1 \approx 2^{-26.99} ) yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( l_{j,19309} = [0.921875, 0.92578113079071044921875] )</td>
<td>( \theta_3 \approx 2^{-27.44} ) ( \eta_3 \approx 2^{-26.90} ) yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( l_{j,19533} = [0.97490406036376953125, 0.97490441799163818359375] )</td>
<td>( \theta_4 \approx 2^{-27.49} ) ( \eta_4 \approx 2^{-26.77} ) yes</td>
<td></td>
</tr>
</tbody>
</table>

Splitting steps when \( m_x < m_y \).
Outline of the talk

- Division via polynomial evaluation
- Generation of an efficient evaluation program
- Validation of the generated evaluation program
- Experimental results
- Concluding remarks
Validation and performance evaluation

- Validation of the complete code:
  - the *Extremal Rounding Tests Set* (D.W. Matula)
  - *TestFloat* package
  - exhaustive tests on mantissa (with fixed result exponent)

- Performances evaluation on ST231 architecture
  - 4-issue VLIW integer processor of ST200 family
  - at most 2 mul. per cycle
  - latencies: addition = 1 cycle, multiplication = 3 cycles
Experimental results

Performances on ST231

<table>
<thead>
<tr>
<th></th>
<th>Nb. of instructions</th>
<th>Latency (# cycles)</th>
<th>IPC</th>
<th>Code size (bytes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rounding to nearest</td>
<td>86</td>
<td>27</td>
<td>3.18</td>
<td>416</td>
</tr>
</tbody>
</table>

- speed-up by a factor of about 1.78 in rounding to nearest compared to the previous implementation (48 cycles)
  - optimized implementation
  - efficient ST200 compiler (*st200cc*)

- high IPC value: confirms the parallel nature of our approach
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Concluding remarks

Contributions

- New approach for the implementation of binary floating-point division
  - bivariate polynomial-based algorithm
  - automatic generation and validation of efficient evaluation program
  - implementation targeted ST231 VLIW integer processor

- Speed-up by a factor of about 1.78 in rounding to nearest compared to the previous implementation

Since then

- Extension to subnormal numbers support
  - implementation in 31 cycles: 4 extra cycles

- Implementation of other functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Latency (# cycles)</th>
<th>IPC</th>
<th>Code size (bytes)</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>square root</td>
<td>21</td>
<td>2.47</td>
<td>276</td>
<td>2.38</td>
</tr>
<tr>
<td>reciprocal</td>
<td>22</td>
<td>2.59</td>
<td>336</td>
<td>1.75</td>
</tr>
<tr>
<td>reciprocal square root</td>
<td>29</td>
<td>2.24</td>
<td>368</td>
<td>2.27</td>
</tr>
</tbody>
</table>