Séminaire – Équipe DALI (Laboratoire ELIAUS – Université de Perpignan)
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Fast and accurate floating-point division on ST231

Algorithm, implementation, and automatic generation and validation

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Context and objectives

Context

- FLIP software library
  - $\longrightarrow$ `http://flip.gforge.inria.fr/`
  - $\longrightarrow$ support for floating-point arithmetic on integer processors
- low latency implementation of binary floating-point division
  - $\longrightarrow$ targets a VLIW integer processor of the ST200 family
- no support of *subnormal* numbers
  - $\longrightarrow$ input/output: ± 0, $\pm\infty$, NaN or *normal* number

Objectives

- faster software implementation (compared to FLIP 0.3)
  - $\longrightarrow$ expose instruction-level parallelism via bivariate polynomial evaluation
- correctly rounded
  - $\longrightarrow$ rounding-to-nearest even

IEEE 754 specification

Let (x, y) be two binary floating-point data:

$$x/y = (-1)^{s_r} \cdot |x|/|y|,$$

with $s_r = s_x$ XOR s_y.

$\|x\|/\|y\|$		$\|y\|$ $+0$	$\|y\|$ normal	$\|y\|$ $+\infty$	$\|y\|$ NaN
$\|x\|$	$+0$	qNaN	$+0$	$+0$	qNaN
$\|x\|$	normal	$+\infty$	$\text{RN}_p(\|x\|/\|y\|)$	$+0$	qNaN
$\|x\|$	$+\infty$	$+\infty$	$+\infty$	qNaN	qNaN
$\|x\|$	NaN	qNaN	qNaN	qNaN	qNaN

Special values for $\text{RN}_p(|x|/|y|)$.

$\Rightarrow$ since $\text{RN}_p(-r) = -\text{RN}_p(r)$, for non special inputs:

$$\text{RN}_p(x/y) = (-1)^{s_r} \cdot \text{RN}_p(|x|/|y|).$$

Notation and assumptions

- Input (x, y): two positive normal numbers

  $\longrightarrow$ precision p, extremal exponents $(e_{\text{min}}, e_{\text{max}})$

$$x = (-1)^{s_x} \cdot m_x \cdot 2^{e_x} \ \text{ with } \begin{cases} s_x \in \{0, 1\} \\ m_x = 1.m_{x,1} \ldots m_{x,p-1} \in [1, 2) \\ e_x \in \{e_{\text{min}}, \ldots, e_{\text{max}}\} \end{cases}$$

- Computation: k-bit unsigned integers

  $\longrightarrow$ register size k

- Example for binary32 format: $(k, p, e_{\text{max}}) = (32, 24, 127)$

Outline of the talk

Division via polynomial evaluation

Generation of an efficient evaluation program

Validation of the generated evaluation program

Experimental results

Concluding remarks

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Division algorithm flowchart

- Definition

$$c = \begin{cases} 1 & \text{if } m_x \geq m_y, \\ 0 & \text{if } m_x < m_y. \end{cases}$$

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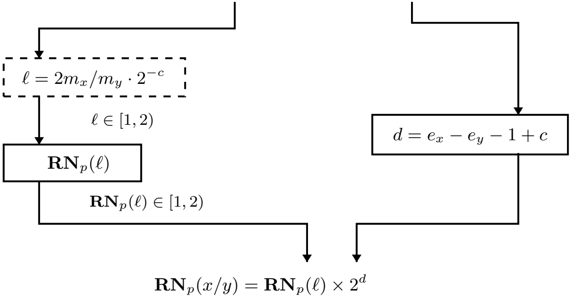
$$x/y = \left(2m_x/m_y \cdot 2^{-c}\right) \times 2^{e_x - e_y - 1 + c}$$



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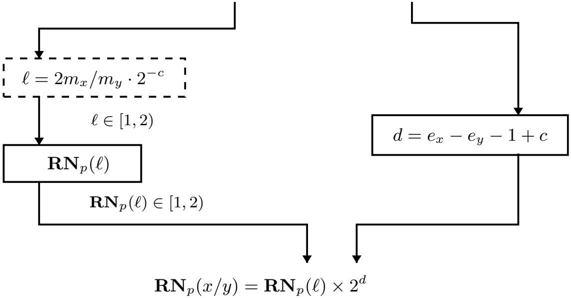

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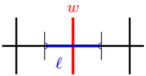
- Polynomial-based methods
  - Agarwal, Gustavson and Schmookler (1999)
    $\longrightarrow$ univariate polynomial evaluation
  - Our approach
    $\longrightarrow$ single bivariate polynomial evaluation

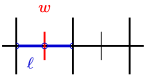
Truncated one-sided approximation

- See for example, Ercegovac and Lang (2004)
- 3 steps
  1. compute $v = (01.v_1 \dots v_{k-2})$ such that

     $$-2^{-p} \leq \ell - v < 0 \text{ that is implied by } |(\ell + 2^{-p-1}) - v| < 2^{-p-1}$$

  2. truncate v after p fraction bits: $w = (01.v_1 \dots v_p 0 \dots 0)$





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How to compute the one-sided approximation v ?

Computation of the one-sided approximation

1. Consider $\ell + 2^{-p-1}$ as the exact result of the function

$$F(s,t) = s/(1+t) + 2^{-p-1},$$

at the points $s^* = 2^{1-c} m_x$ and $t^* = m_y - 1$:

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How to ensure that $|(\ell + 2^{-p-1}) - v| < 2^{-p-1}$?

Sufficient error bounds

- Since by triangular inequality

$$|(\ell + 2^{-p-1}) - v| \leq \mu \cdot \epsilon_{\mathsf{approx}} + \epsilon_{\mathsf{eval}}$$

with

$$\mu = \max\{s^*\} = \max\{2^{1-c} m_x\} = (4 - 2^{3-p})$$

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- One has to ensure

$$\mu \cdot \epsilon_{\text{approx}} + \epsilon_{\text{eval}} < 2^{-p-1}$$

- Sufficient conditions can be obtained

$$\epsilon_{\text{approx}} < 2^{-p-1}/\mu \quad \text{and} \quad \epsilon_{\text{eval}} < 2^{-p-1} - \mu \cdot \epsilon_{\text{approx}}$$

Implementation steps

1. determine the minimal degree δ for the polynomial approximant a
2. compute the polynomial approximant a such that
$$\epsilon_{\text{approx}} < 2^{-p-1}/\mu$$
3. find an efficient evaluation program $\mathcal{P}$ such that
$$\epsilon_{\text{eval}} < 2^{-p-1} - \mu \cdot \epsilon_{\text{approx}}$$
4. validate the evaluation program

$\Rightarrow$ implemented using *Sollya* (steps 1 and 2) and *Gappa* (step 4)

Outline of the talk

Automatic generation of an efficient evaluation program

- Evaluation program $\mathcal{P}$ = main part of the full software implementation
  $\longrightarrow$ dominates the cost

- By efficient, one means an evaluation program that
  - $\longrightarrow$ reduces the evaluation latency
  - $\longrightarrow$ reduces the number of multiplications
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  $\longrightarrow$ 4-issue VLIW integer processor with at most 2 mul. per cycle
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Which evaluation program to evaluate the polynomial $P(s,t)$?

Example for the binary32 implementation: $(k, p) = (32, 24)$

$$P(s,t) = 2^{-p-1} + s \cdot \sum_{i=0}^{10} a_i t^i$$

- Horner's scheme: $(3+1) \times 11 = 44$ cycles
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  - $\longrightarrow$ a last multiplication by s
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- ...

We can do much better.

- But how to explore the solution space and choose an efficient evaluation program ?
  - $\longrightarrow$ interest of automatic generation

Efficient evaluation tree generation

- Similar to Harrison, Kubaska, Story and Tang (1999)
- Assumption
  - $\longrightarrow$ unbounded parallelism
  - $\longrightarrow$ latencies of arithmetic operators: $+$ and $\times$

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- Two sub-steps
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     $$\text{ie. } \tau = 3 \times \lceil \log_2(\deg(P)) \rceil + 1$$

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  $\Rightarrow$ if no tree satisfies τ then increase τ and restart
- Number of evaluation trees = extremely large $\longrightarrow$ several filters

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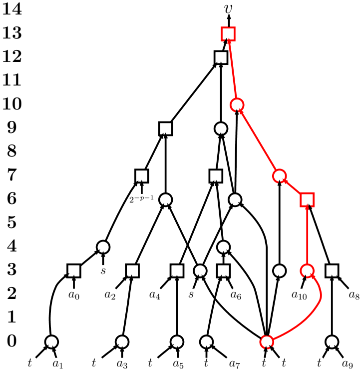

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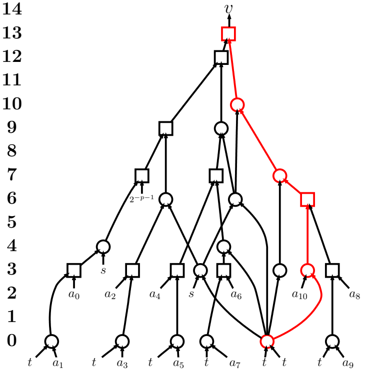
- first target latency $\tau = 13$
  $\longrightarrow$ no tree found
- second target latency $\tau = 14$
  $\longrightarrow$ obtained in about 10 sec.
- distribute the multiplication by s
  $\longrightarrow$ otherwise: 18 cycles
- too difficult to find such tree by hand



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- Label evaluation trees by appropriate arithmetic operator: $+$ or $-$
- If the sign of an intermediate value changes when the input varies then the evaluation tree is rejected

  $\Rightarrow$ implementation with certified interval arithmetic (MPFI)

Practical scheduling checking

- Schedule the evaluation trees on a simplified model of a real target architecture
  - $\longrightarrow$ operator costs, nb. issues, constraints on operators
  - $\longrightarrow$ no syllables constraint

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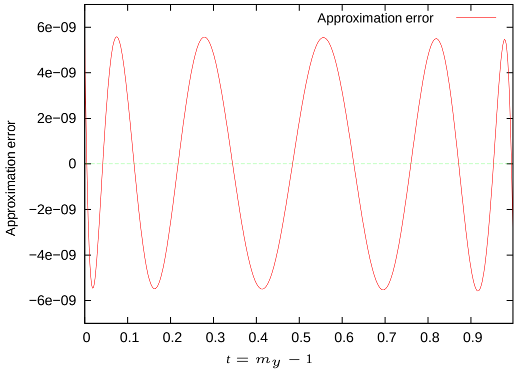
- Schedule the evaluation trees on a simplified model of a real target architecture
  - $\longrightarrow$ operator costs, nb. issues, constraints on operators
  - $\longrightarrow$ no syllables constraint
- Check if no increase of latency in practice compared to the latency on unbounded parallelism
  - $\Rightarrow$ if practical latency $>$ theoretical latency then the evaluation tree is rejected
  - $\Rightarrow$ implementation using naive list scheduling algorithm is enough

Outline of the talk

Example for the binary32 implementation: $(k, p) = (32, 24)$

- Approximation of $1/(1+t)$ by truncated Remez' polynomial of degree 10



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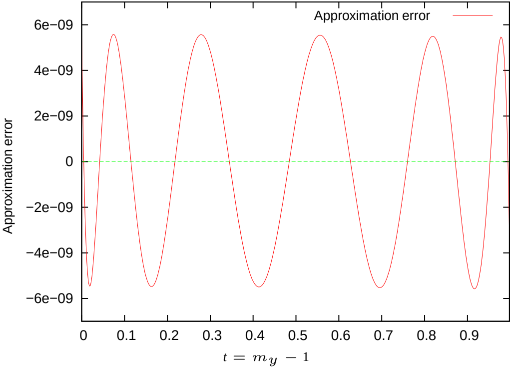


$$\epsilon_{\text{approx}} \leq 2^{-27.41\ldots} \approx 6.0 \text{ e-9} \quad < \quad 2^{-25}/(4 - 2^{-21}) \approx 7.4 \text{ e-9}$$

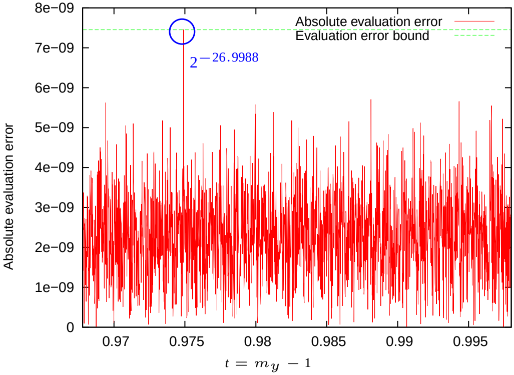
- Deduction of the evaluation error bound from ϵ_{approx}

$$\epsilon_{\text{eval}} < 2^{-25} - (4 - 2^{-21}) \cdot 2^{-27.41\ldots} \approx 2^{-26.9999\ldots} \approx 7.4 \text{ e-9}.$$

Example for the binary32 implementation: $(k, p) = (32, 24)$

- Case 1: $m_x \geq m_y \rightarrow$ condition satisfied
- Case 2: $m_x < m_y \rightarrow$ condition not satisfied

ie. $s^* = 3.935581684112548828125$ and $t^* = 0.974904417991638183594375$



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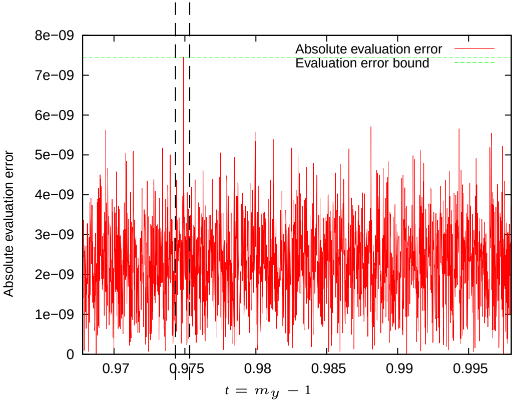

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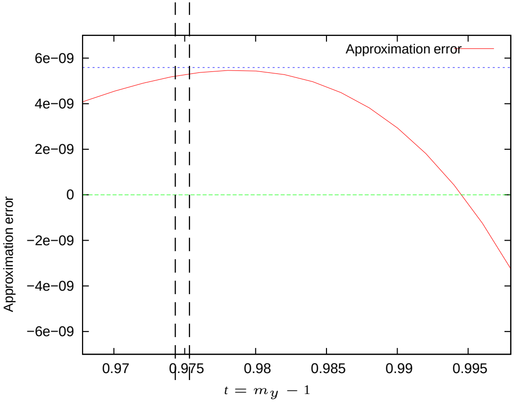

1. determine an interval $\mathcal{I}$ around this point
2. compute ϵ_{approx} over $\mathcal{I}$
3. determine an evaluation error bound η
4. check if $\epsilon_{\text{eval}} < \eta$?

Evaluation program validation strategy

- Find a splitting of the input interval into n subinterval(s) $\mathcal{T}^{(i)}$, and check that
$$\mu \cdot \epsilon_{\text{approx}}^{(i)} + \epsilon_{\text{eval}}^{(i)} < 2^{-p-1}$$
on each subinterval.

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- Implementation of the splitting by dichotomy
  - for each $\mathcal{T}^{(i)}$
    1. compute a certified approximation error bound $\epsilon_{\text{approx}}^{(i)}$
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  $\Rightarrow$ if this bound is not satisfied, $\mathcal{T}^{(i)}$ is split up into 2 subintervals
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- Example of binary32 implementation

  $\longrightarrow$ launched on a 64 processor grid

  $\longrightarrow$ 36127 subintervals found in several hours (≈ 5h.)

Evaluation program validation strategy

* Does the condition

$$\mu \cdot \epsilon_{\text{approx}}^{(i)} + \epsilon_{\text{eval}}^{(i)} < 2^{-p-1}$$

hold for $i \in \{1, \ldots, n\}$?

Depth	Subintervals	$\epsilon_{\text{approx}}^{(\cdot)}(a) \leq$	$\epsilon_{\text{eval}}^{(\cdot)}(\mathcal{P}) <$	*
1	$\mathrm{I}_{1,1} = [2^{-23}, 1 - 2^{-23}]$	$\theta_1 \approx 2^{-27.41}$	$\eta_1 \approx 2^{-26.99}$	no
2	$\mathrm{I}_{2,1} = [2^{-23}, 0.5 - 2^{-23}]$	$\theta_2 \approx 2^{-27.41}$	$\eta_2 \approx 2^{-26.99}$	yes
	$\mathrm{I}_{2,2} = [0.5, 1 - 2^{-23}]$	$\theta_1 \approx 2^{-27.41}$	$\eta_1 \approx 2^{-26.99}$	no
$\ldots$				
j	$\mathrm{I}_{j,1} = [2^{-23}, 0.5 - 2^{-23}]$	$\theta_2 \approx 2^{-27.41}$	$\eta_2 \approx 2^{-26.99}$	yes
	$\mathrm{I}_{j,2} = [0.5, 0.75 - 2^{-23}]$	$\theta_1 \approx 2^{-27.41}$	$\eta_1 \approx 2^{-26.99}$	yes
	$\mathrm{I}_{j,19309} = [0.921875, 0.92578113079071044921875]$	$\theta_3 \approx 2^{-27.44}$	$\eta_3 \approx 2^{-26.90}$	yes
	$\mathrm{I}_{j,19533} = [0.97490406036376953125, 0.97490441799163818359375]$	$\theta_4 \approx 2^{-27.49}$	$\eta_4 \approx 2^{-26.77}$	yes

Splitting steps when $m_x < m_y$.

Outline of the talk

Validation and performance evaluation

- Validation of the complete code:
  - → the *Extremal Rounding Tests Set* (D.W. Matula)
  - → *TestFloat* package
  - → exhaustive tests on mantissa (with fixed result exponent)
- Performances evaluation on ST231 architecture
  - → 4-issue VLIW integer processor of ST200 family
  - → at most 2 mul. per cycle
  - → latencies: addition = 1 cycle, multiplication = 3 cycles

Experimental results

Performances on ST231

	Nb. of instructions	Latency (# cycles)	IPC	Code size (bytes)
rounding to nearest	86	27	3.18	416

- speed-up by a factor of about 1.78 in rounding to nearest compared to the previous implementation (48 cycles)
  - optimized implementation
  - efficient ST200 compiler (`st200cc`)
- high IPC value: confirms the parallel nature of our approach

Outline of the talk

Concluding remarks

Contributions

- New approach for the implementation of binary floating-point division
  - $\longrightarrow$ bivariate polynomial-based algorithm
  - $\longrightarrow$ automatic generation and validation of efficient evaluation program
  - $\longrightarrow$ implementation targeted ST231 VLIW integer processor
- Speed-up by a factor of about 1.78 in rounding to nearest compared to the previous implementation

Since then

- Extension to subnormal numbers support
  - $\longrightarrow$ implementation in 31 cycles: 4 extra cycles
- Implementation of other functions

	Latency (# cycles)	IPC	Code size (bytes)	Speed-up
square root	21	2.47	276	2.38
reciprocal	22	2.59	336	1.75
reciprocal square root	29	2.24	368	2.27