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Automatic Generation of Fast and Certified Code for Polynomial Evaluation

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Joint work with Christophe Mouilleron (LIP, ENS Lyon)

Motivation

- Embedded systems are ubiquitous
  - microprocessors dedicated to one or a few specific tasks
  - satisfy constraints: area, energy consumption, conception cost

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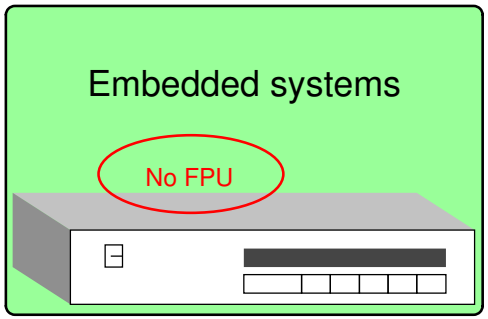

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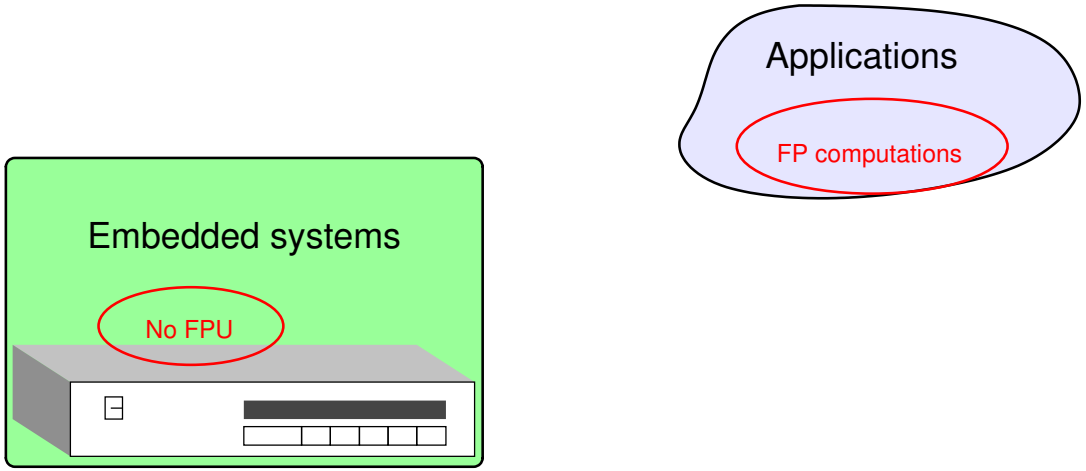

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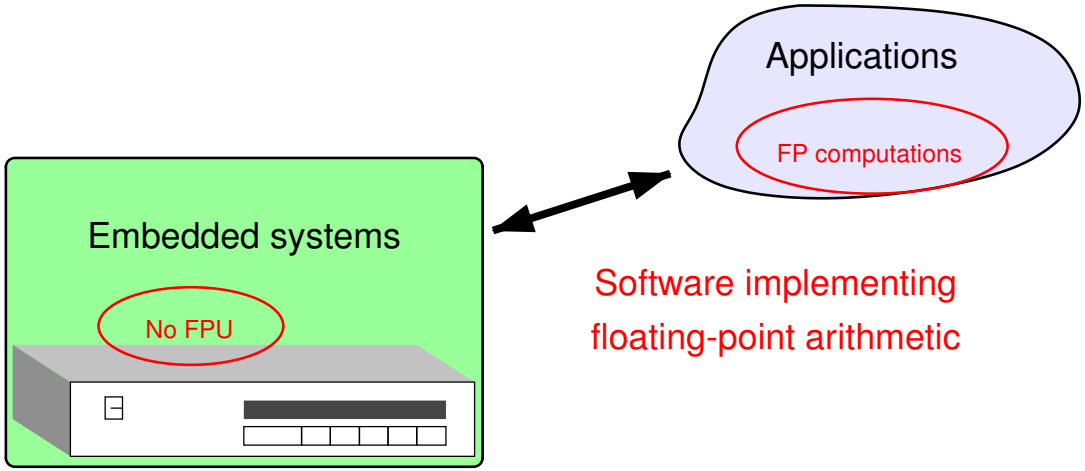

- Highly used in audio and video applications
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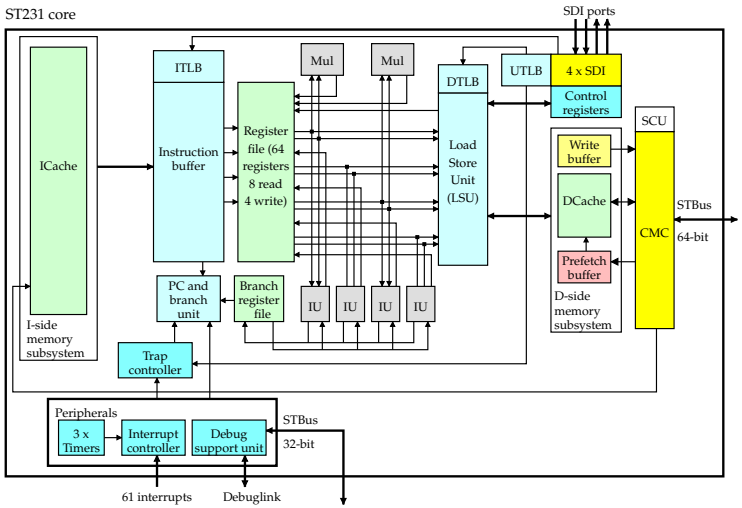
On the one side: the IEEE 754-2008 standard, ...

- Definition of IEEE floating-point arithmetic
  - floating-point formats: single precision, double precision, ...
  - special values: ± 0, $\pm\infty$, NaN
  - 4 rounding modes: to nearest even, upward, downward, and toward zero
  - mathematical function behavior
    - $\rightarrow$ special input (ex: $\sqrt{-0} = -0$)
    - $\rightarrow$ requires / recommends correct rounding
- Motivation:
  - make computations reproducible
  - and make results architecture-independent

... on the other side: the ST231 processor



- 4-issue VLIW 32-bit integer processor
  - $\longrightarrow$ no FPU
- Parallel execution unit
  - 4 integer ALUs
  - 2 pipelined multipliers $32 \times 32 \longrightarrow 32$
- Latencies: ALU $=$ 1 cycle / Mul $=$ 3 cycles

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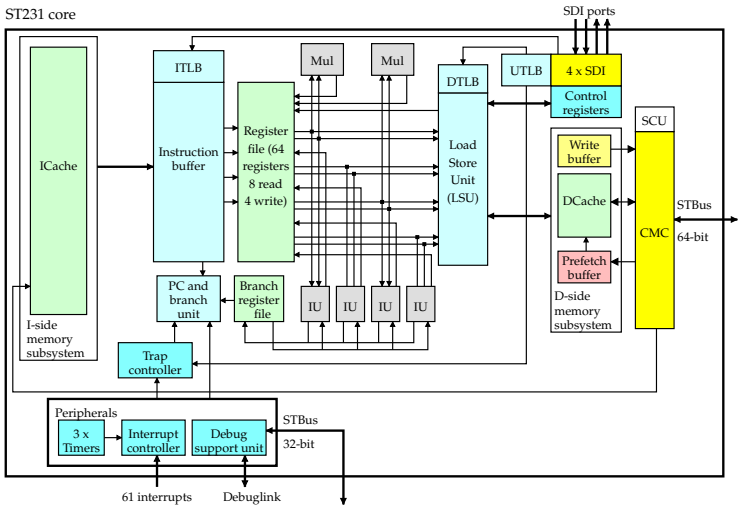

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- VLIW (Very Long Instruction Word)
  - $\longrightarrow$ instructions grouped into bundles
  - $\longrightarrow$ Instruction-Level Parallelism (ILP) explicitly exposed by the compiler

Towards the generation of fast and certified codes

- This work takes mainly part in the context of the development of FLIP
  - ↪ software support for binary32 floating-point arithmetic on integer processors

- Underlying problem: development “by hand”
  - long and tedious, error prone
  - new target ? new floating-point format ?

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  } automation and certification

- Current challenge: tools and methodologies for the automatic generation of efficient and certified programs
  - optimized for a given format, for the target architecture

Towards the generation of fast and certified codes

- Some works on code generation and transformation:
  - ↪ code generators: hardware (FloPoCo) and software (Sollya, Metalibm)
  - ↪ code transformation for increasing numerical accuracy [Martel, 2009]
- LEMA project [Lefèvre et al., 2010]: language and library
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Can we teach computers to write fast libraries?

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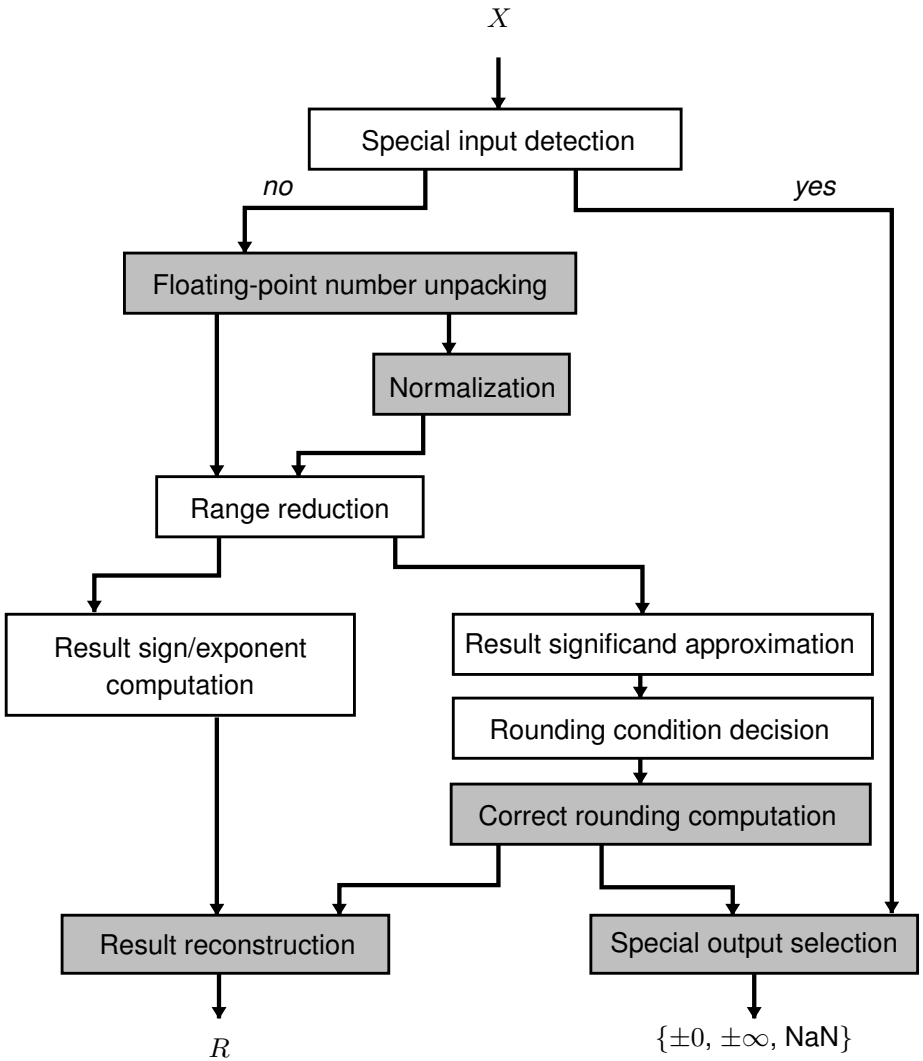
Can we teach computers to write fast libraries?

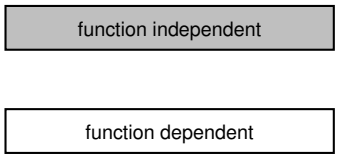
- Our tool: CGPE (Code Generation for Polynomial Evaluation)

*In the particular case of **polynomial evaluation**, can we teach computers to write **fast and certified** codes, for a given target and optimized for a given format?*

  - ↪ adding a systematic certification step

Basic blocks for implementing correctly-rounded operators

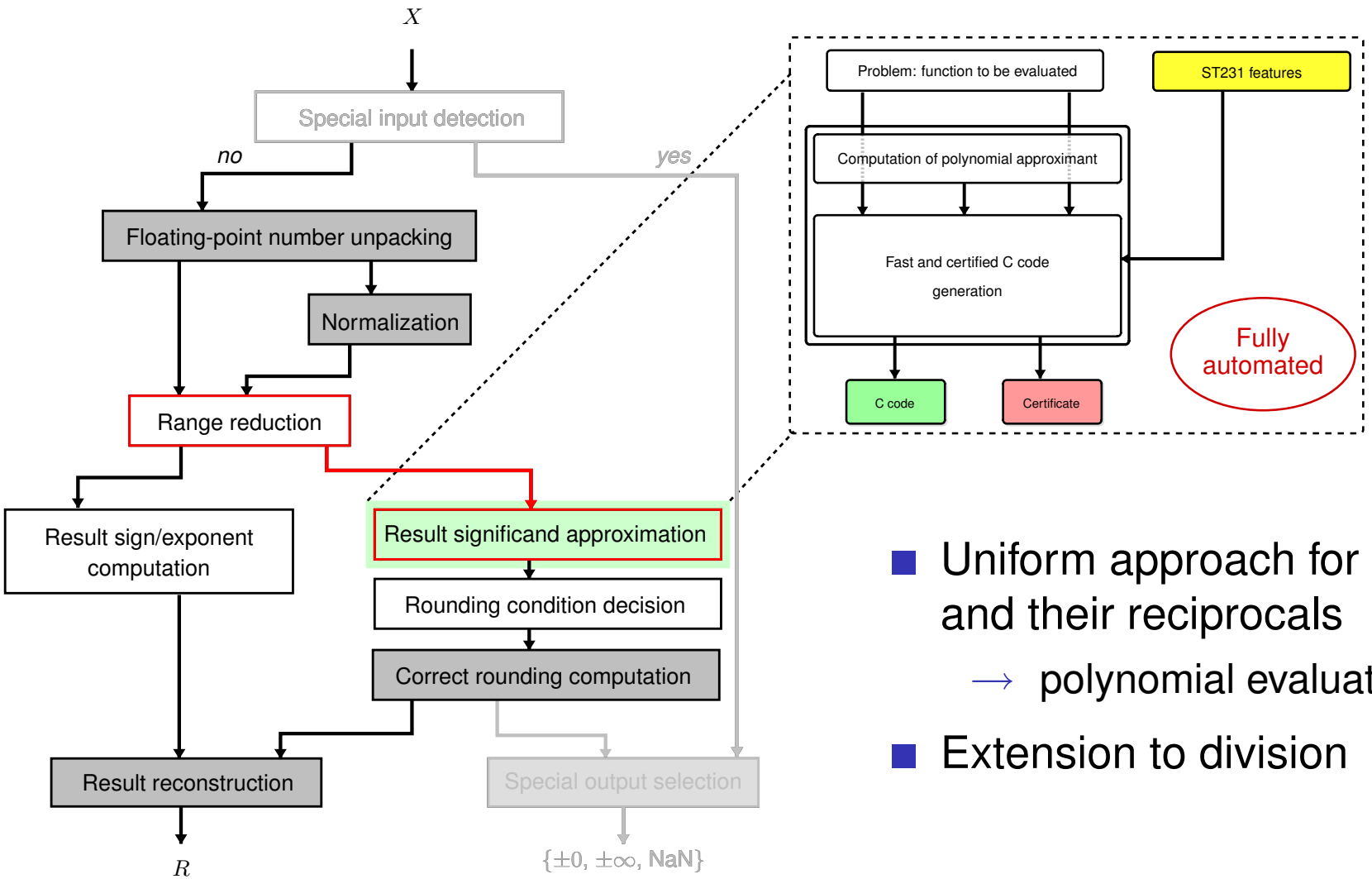




Objectives

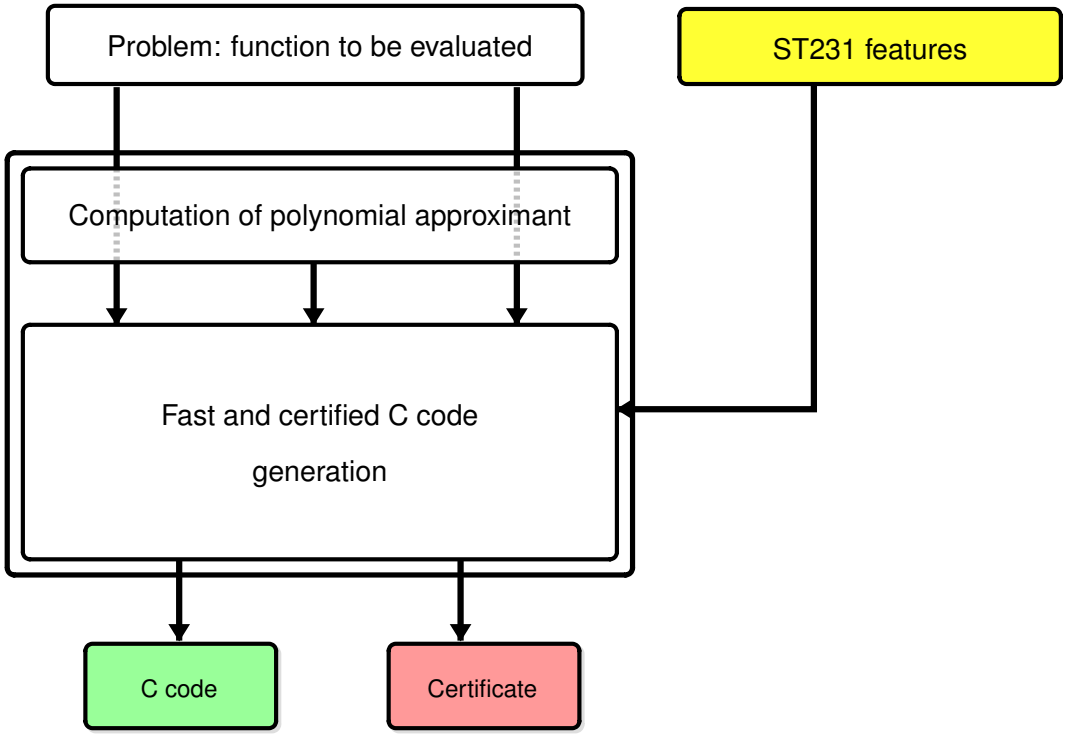
- → Low latency, correctly-rounded implementations
- → ILP exposure

Basic blocks for implementing correctly-rounded operators



- Uniform approach for *n*th roots and their reciprocals
  - $\longrightarrow$ polynomial evaluation
- Extension to division

Flowchart for generating efficient and certified C codes



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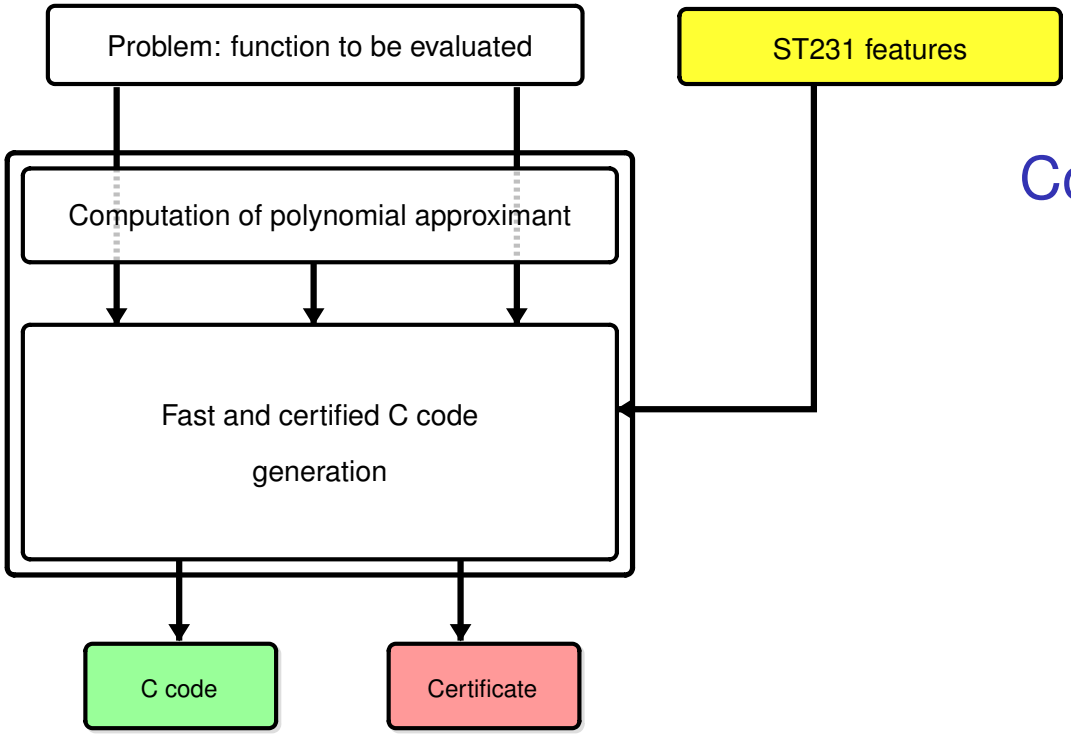

Constraints

- Accuracy of approximant and C code

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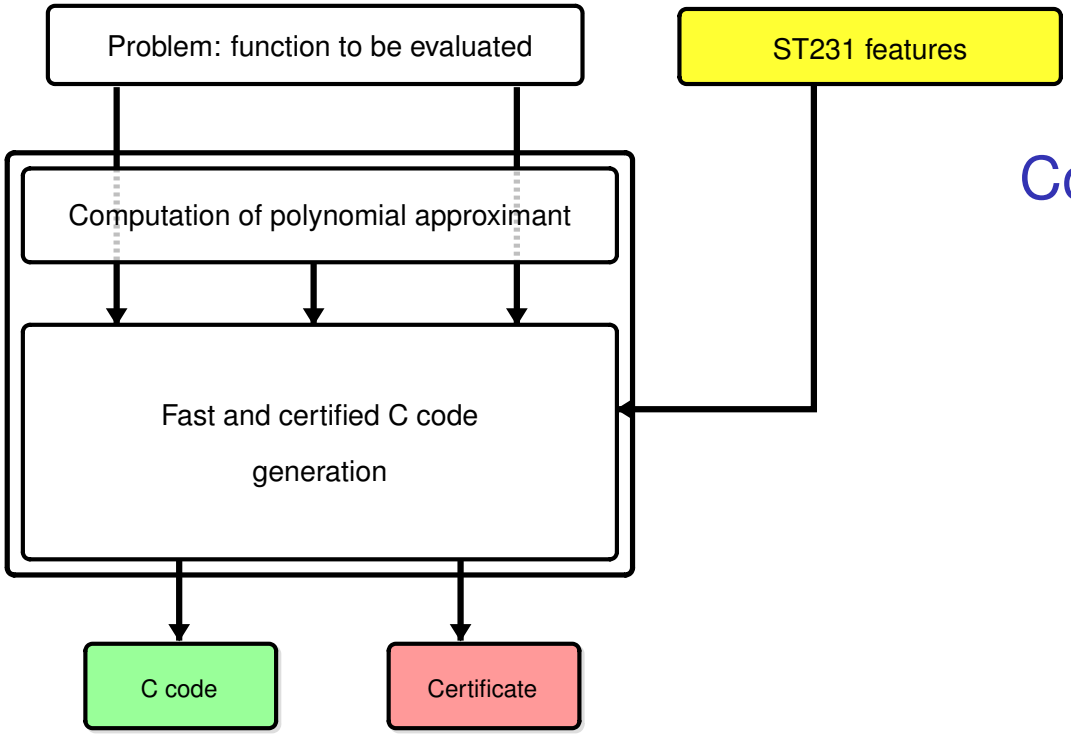

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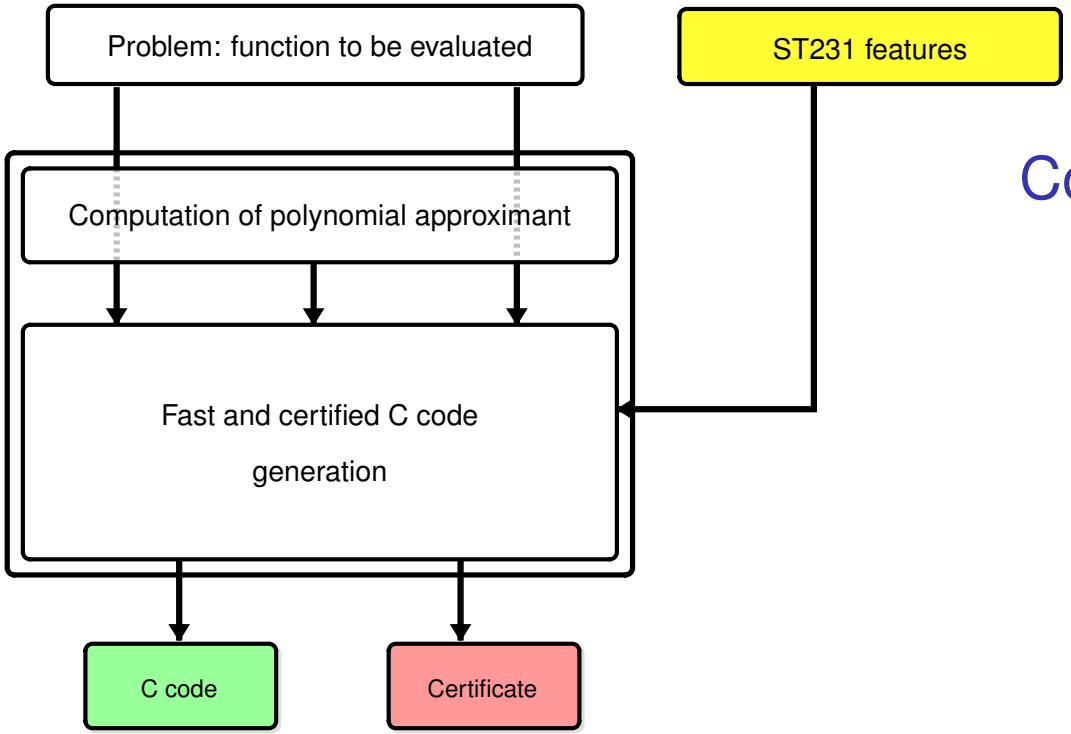

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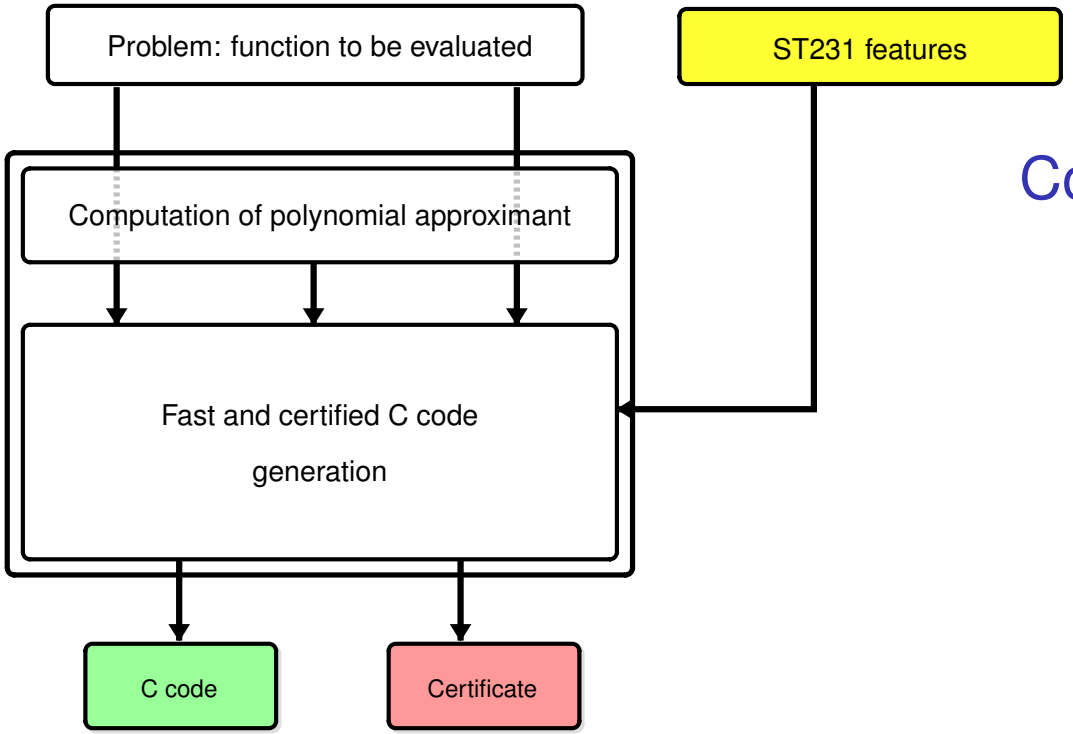

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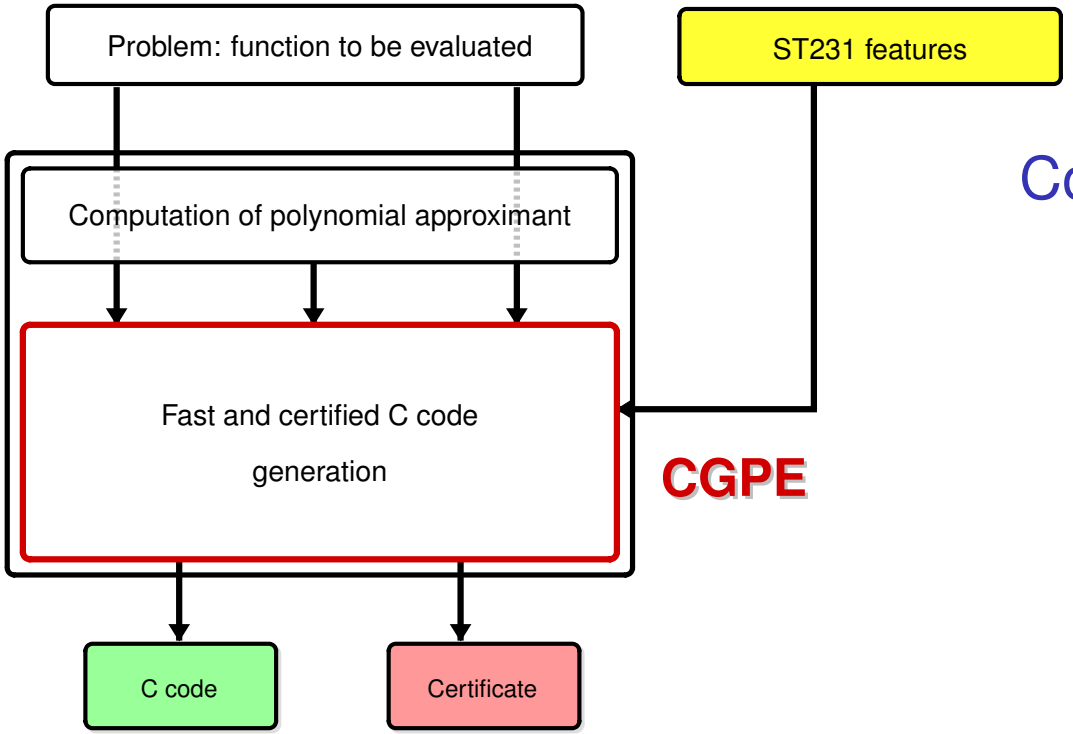

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  - ?

Outline of the talk

1. Background on polynomial evaluation
2. The CGPE tool
3. Experimental results
4. Conclusion

Outline of the talk

Our objective

- Compute fast and certified schemes for evaluating a polynomial $P(x, y) = \alpha + y \cdot a(x)$
  - $\longrightarrow$ using only additions and multiplications
  - $\longrightarrow$ reducing the evaluation latency on unbounded parallelism

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- Two families of algorithms
  - algorithms with coefficient adaptation: Knuth and Eve (60's), Paterson and Stockmeyer (1973), ...
    - $\longrightarrow$ ill-suited in the context of fixed-point arithmetic
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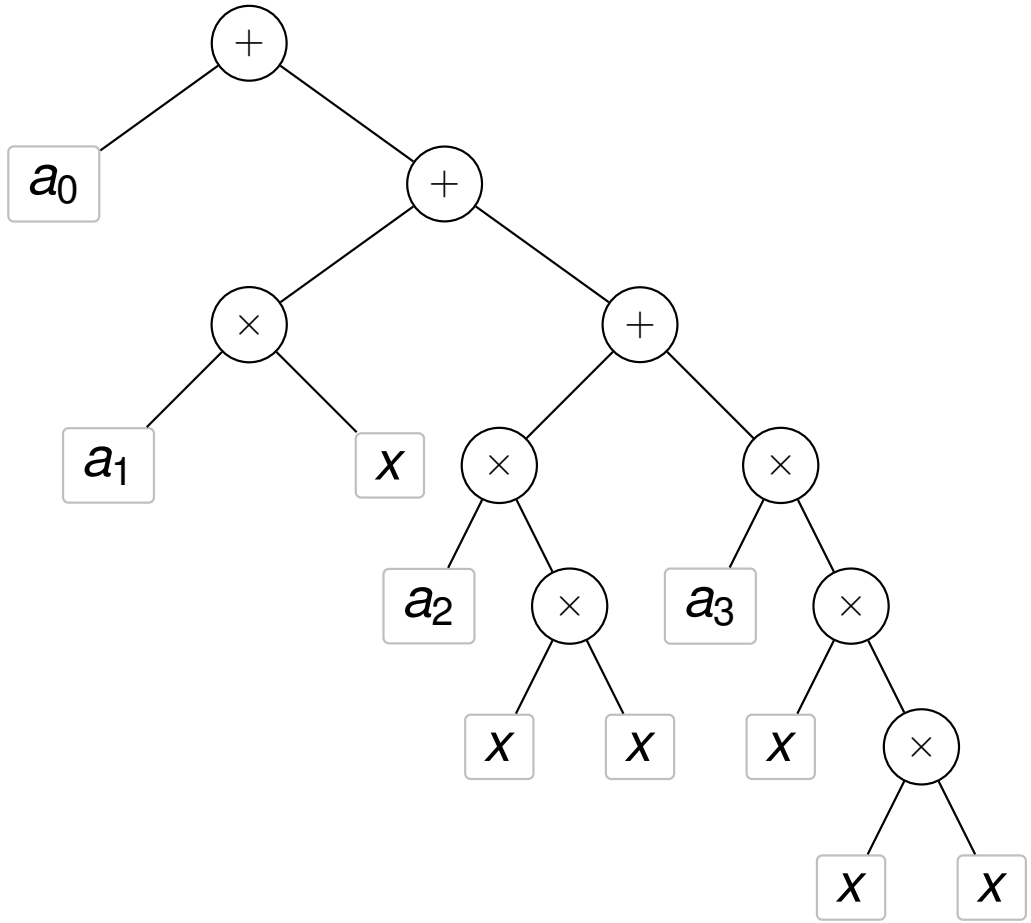
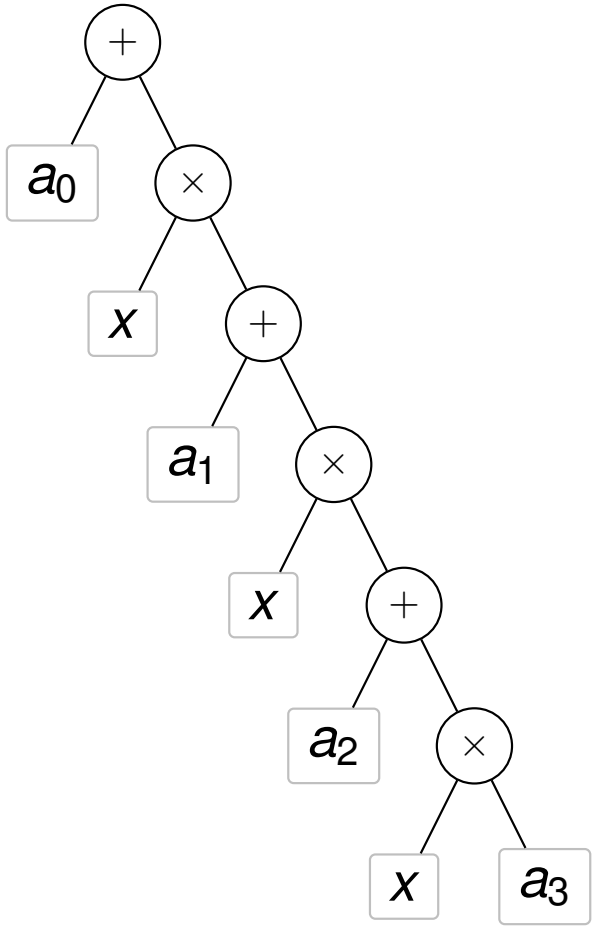
Classical evaluation schemes

- Naive evaluation
  - ↪ 3 additions
    5 multiplications
  - ↪ latency: 12 cycles



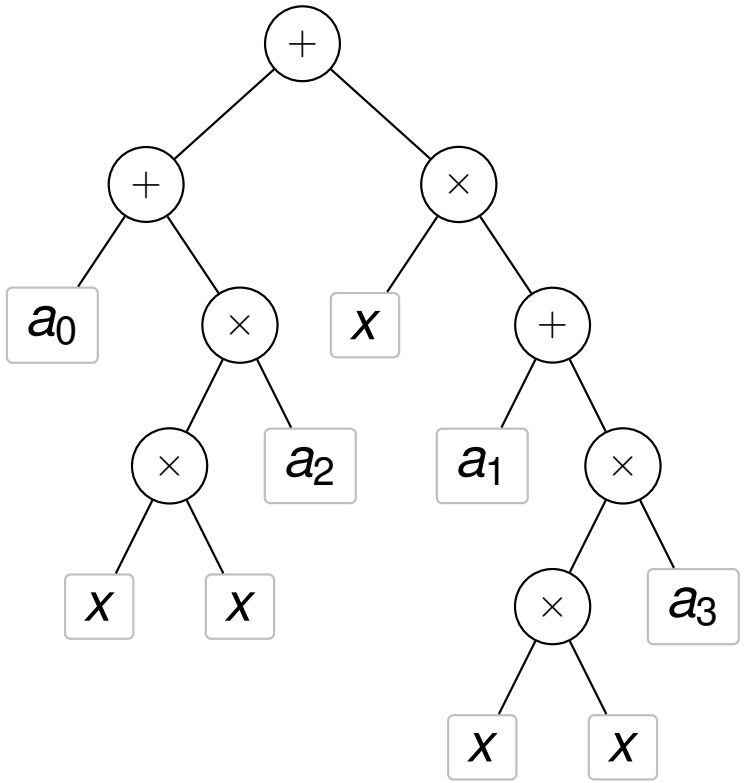
Classical evaluation schemes

- Horner's rule
  - ↪ 3 additions
    3 multiplications
  - ↪ latency: 12 cycles
  - ⊕ optimal in terms of multiplication number
    [Pan, 1966], [Borodin, 1971],
  - ⊖ fully sequential



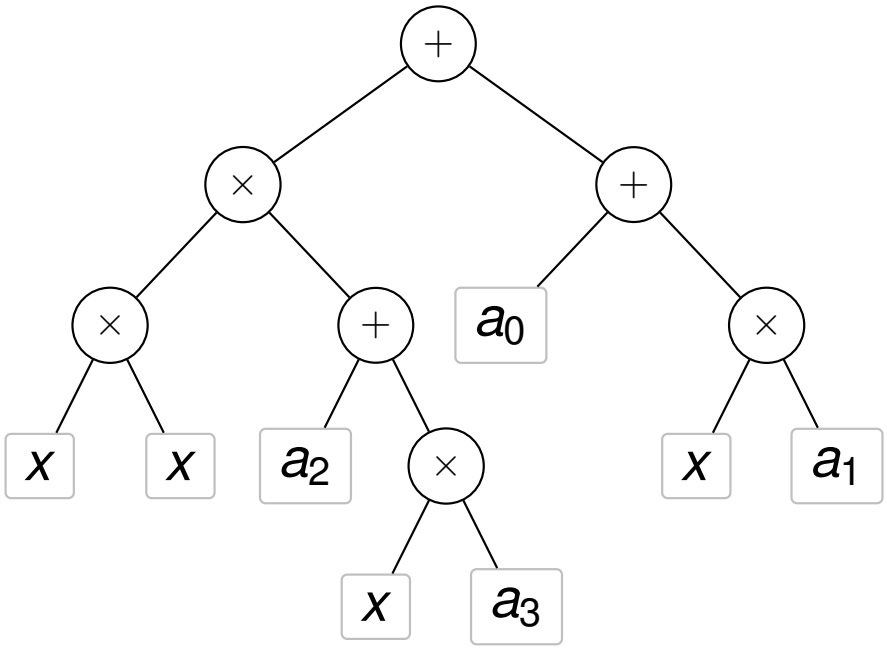
Classical evaluation schemes

- Second-order Horner's rule
  - ↪ 3 additions
    4 multiplications
  - ↪ latency: 11 cycles
  - ⊕ some ILP exposure
  - ⊖ subparts evaluated in a fully sequential way: at most 2 ways used



Classical evaluation schemes

- Estrin's rule
  - ↪ 3 additions
    4 multiplications
  - ↪ latency: 8 cycles
  - ⊕ "divide and conquer" strategy
  - ⊕ more ILP exposure



Definition of evaluation schemes

Mathematical expression $a_0 + a_1 \cdot x + a_2 \cdot x^2$ 1

Common *parenthesizations* $(a_0 + (a_1 \cdot x)) + (a_2 \cdot (x \cdot x))$ 1

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commutativity
associativity
distributivity

Feasible *parenthesizations* $(a_0 + (a_1 \cdot x)) + (a_2 \cdot (x \cdot x))$ 160

$(a_0 + (a_1 \cdot x)) + ((a_2 \cdot x) \cdot x)$

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$((a_1 + (a_2 \cdot x)) \cdot x) + a_0 \quad \ldots$

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Feasible *parenthesizations* $[\ldots]$ 160

$+$ and $\times$ are only commutative in fixed-point arithmetic

Evaluation schemes

$(a_0 + (a_1 \cdot x)) + (a_2 \cdot (x \cdot x))$
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7

Remarks on polynomial evaluation

- There are several other schemes for evaluating a polynomial $a(x)$
  - can be adapted for bivariate polynomial $P(x, y) = \alpha + y \cdot a(x)$
- Constant number of $+$, while number of $\times$ is non-constant
  - reducing the latency $\Leftrightarrow$ increasing the number of $\times$ to expose ILP
  - trade-off latency / number of multiplications

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  - different theoretical error bounds
  - difference between numerical quality in practice [Revy, 2006]

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$\rightsquigarrow$ We need a tool for exploring the space of evaluation schemes.

How many schemes for evaluating a polynomial?

n	$\mu_n \to a(x)$	$\mu'_n \to \alpha + y \cdot a(x)$		
1	1	10		
2	7	481		
3	163	88384		
4	11602	57363910		
5	2334244	122657263474		
6	1304066578	829129658616013		
7	1972869433837	17125741272619781635		
8	8012682343669366	1055157310305502607244946		
9	86298937651093314877	190070917121184028045719056344		
10	2449381767217281163362301	985436908485543809474905225911916 72		

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8	8012682343669366	10551573103055026072449 46	23	
9	86298937651093314877	19007091712118402804571905 6344	46	
10	2449381767217281163362301	98543690848554380947490522591191672	98	

- Two well-known special cases
  - the number of evaluation schemes for x^n [Wedderburn, Etherington]

$$w_n \sim \frac{\eta \xi^n}{n^{3/2}} \text{ or } \begin{cases} \xi \approx 2.48325 \\ \eta \approx 0.31877 \end{cases} \text{ [Otter, 1948],}$$

How many schemes for evaluating a polynomial?

n	$\mu_n \to a(x)$	$\mu'_n \to \alpha + y \cdot a(x)$	w_n	$(2n-1)!!$
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2	7	481	1	3
3	163	88384	1	15
4	11602	57363910	2	105
5	2334244	122657263474	3	945
6	1304066578	829129658616013	6	10395
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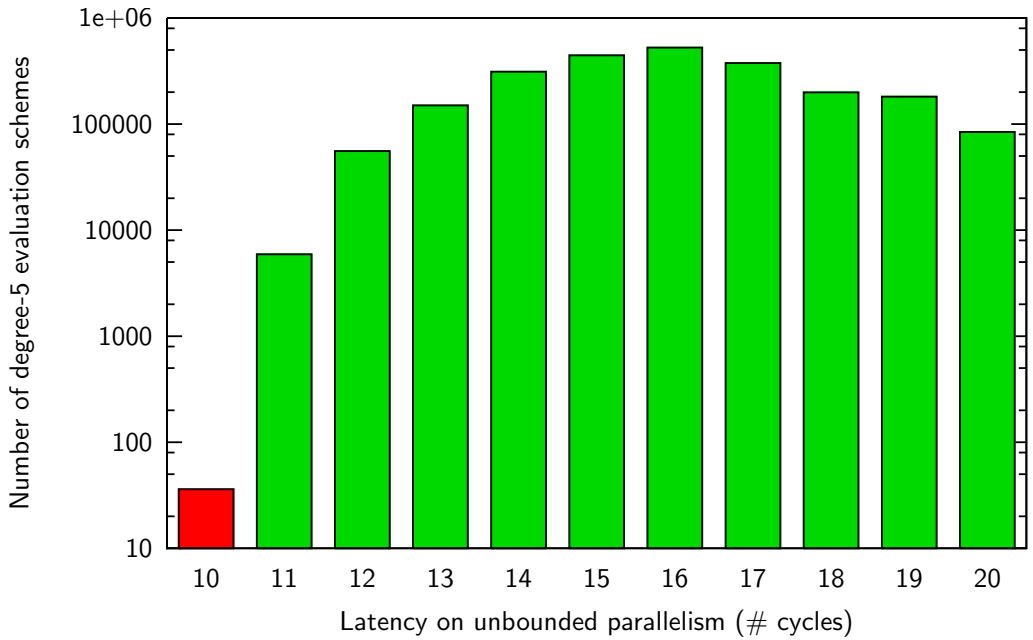
  - the number of evaluation schemes for $\sum_{i=1}^{n} a_i$ est $(2n-1)!! \sim \sqrt{2}\left(\frac{2n}{e}\right)^n$.

Schemes of low evaluation latency

- What is the latency of degree-5 evaluation schemes?



- Total number of schemes: 2334244
  - ⇝ minimal latency for degree-5 univariate polynomial: 10 cycles
  - ⇝ number of schemes of minimal latency: 36

Outline of the talk

Overview of CGPE and related works

- Goal of CGPE [Mouilleron and Revy, 2011]: automate the design of fast and certified C codes for evaluating univariate/bivariate polynomials
  - in fixed-point arithmetic
  - by using the target architecture features (as much as possible)

- Remarks:
  - $\rightsquigarrow$ fast = that reduces the evaluation latency on a given target
  - $\rightsquigarrow$ certified = we can bound the error entailed by the evaluation within the given target's arithmetic

Overview of CGPE and related works

- Some related works
  - [Cheung et al., 2005] and [Lee and Villasenor, 2009]: methodology for implementing automatically mathematical function in a given precision
    - ⊖ based on small degree polynomial evaluation using Horner's → no ILP
  - [Harrison et al., 1999]: method for generating optimal evaluation scheme to evaluate univariate polynomials on Itanium® using `fma`
    - ⊖ ST231 has only addition and multiplication, but no `fma`
  - [Green, 2002]: *brute force* method for generating polynomial evaluation schemes using at best SIMD instructions of the processor of PlayStation® 2
    - ⊖ objective: generation at compile-time → *brute force* method is unfeasible

Global architecture of CGPE

- Input of CGPE
  1. polynomial coefficients and variables: value intervals, fixed-point format, ...
  2. set of criteria: maximum error bound and bound on latency (or the lowest)
  3. delay of one of the variable
  4. some architectural constraints: operator cost, parallelism, ...

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  4. some architectural constraints: operator cost, parallelism, ...
- CGPE works in two steps:
  1. computation of evaluation schemes: reducing evaluation latency on unbounded parallelism and exposing as much ILP as possible
  2. selection among the generated schemes, according to different criteria:
     - evaluation using only unsigned fixed-point arithmetic
     - scheduling feasible on ST231
     - evaluation error bound satisfying the required error bound
- At the end: CGPE automatically writes C codes with accuracy certificates

Heuristics in DAG set computation

- Determination of the minimal target latency on unbounded parallelism
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- Non exhaustive computation of evaluation schemes
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  - limitation to some splittings: evaluation of high and low parts separately
  - restriction to N schemes at each step of the computation
- At the end of the computation: set of DAG evaluating the input polynomial and satisfying the latency constraint on unbounded parallelism.

Filters for adding numerical constraints

1. Arithmetic operator choice
   - ensure that all intermediate variables are of constant sign
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2. Scheduling on a simplified model of the target (like the ST231)
   - constraints of architecture: cost of operators, instruction bundling, ...
   - delays on variables

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   - ensure that all intermediate variables are of constant sign
   - avoid an extra cost due to sign handling / gain 1 bit of accuracy
2. Scheduling on a simplified model of the target (like the ST231)
   - constraints of architecture: cost of operators, instruction bundling, ...
   - delays on variables
3. Evaluation error bound checking
   - straightline polynomial evaluation program
   - “C code certification” using Gappa
     - $\rightsquigarrow$ we can bound the evaluation error in integer arithmetic

Outline of the talk

Timings for certified code generation

	$x^{1/2}$	$x^{-1/2}$	$x^{1/3}$	$x^{-1/3}$	$\log_2(1+x)$	$\frac{1}{\sqrt{1+x^2}}$	$\frac{\exp(1+x)}{1+x}$
Degree (d_x, d_y)	(8,1)	(9,1)	(8,1)	(9,1)	(6,0)	(7,0)	(10,0)
Target / Minimal latency	13 / 13	13 / ?	16 / 16	16 / 16	10 / 11	10 / 11	13 / 13
Achieved latency	13	14	16	16	11	11	13
Scheme computation	195ms [50]	73ms [50]	26s [50]	25s [50]	17ms [50]	10ms [50]	40ms [50]
Arithmetic operator choice	3ms [35]	3ms [29]	7ms [30]	11ms [26]	1ms [2]	2ms [12]	3ms [27]
Scheduling checking	16s [11]	1m33s [1]	43ms [30]	439ms [24]	2ms [1]	64ms [5]	49s [5]
Certification (Gappa)	10s [11]	1s [1]	27s [30]	27s [24]	230ms [1]	1s [5]	7s [4]
Total time ($\approx$)	27s	1m35s	55s	53s	1s	2s	57s

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1. Impact of the target latency on the first step of the generation

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1. Impact of the target latency on the first step of the generation
2. What may dominate the cost: scheduling and certification using Gappa

Timings for certified code generation

	$x^{1/2}$	$x^{-1/2}$	$x^{1/3}$	$x^{-1/3}$	$\log_2(1+x)$	$\frac{1}{\sqrt{1+x^2}}$	$\frac{\exp(1+x)}{1+x}$
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1. Impact of the target latency on the first step of the generation
2. What may dominate the cost: scheduling and certification using Gappa
3. Optimality of some generated codes, in terms of evaluation latency

Outline of the talk

Conclusions

- Code generation for fast and certified polynomial evaluation
  - in fixed-point arithmetic
  - methodologies and tools to automate polynomial evaluation implementation
  - heuristics and techniques for generating quickly fast and certified C codes
  - implemented in the tool CGPE (Code Generation for Polynomial Evaluation)

`http://cgpe.gforge.inria.fr/`

- Speed-up significantly the development time of mathematical library
  - CGPE: allows to write and certify automatically $\approx 50\ \%$ of the codes of FLIP

Current work and perspectives

- Current work
  - precomputation in order to help the DAG set computation in choosing the appropriate splittings: the ones leading to DAGs with optimal latency on unbounded parallelism
  - earlier DAG elimination, by checking accuracy during generation step

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  - precomputation in order to help the DAG set computation in choosing the appropriate splittings: the ones leading to DAGs with optimal latency on unbounded parallelism
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- Perspectives
  - extend CGPE to handle floating-point arithmetic,
  - make CGPE more general to tackle other problems, like evaluation of a polynomial at a matrix point.

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